

Single Field Baryogenesis and the Scale of Inflation

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In the context of inflationary cosmology, we discuss a minimal baryogenesis scenario in which the resulting baryon to entropy ratio is determined by the amplitude of the anisotropies of the cosmic microwave background. The model involves a new $SU(2)_L$ scalar field which generates a Dirac neutrino mass and which is excited by quantum fluctuations during inflation, yielding a CP-violating phase. During the scalar field decay after inflation, an asymmetry in the left-handed neutrino number is generated, which then converts to a net baryon asymmetry via sphalerons. A lower limit on the expected initial value of the scalar field translates to a lower limit on the baryon to entropy ratio (which also depends on the Dirac neutrino Yukawa coupling). Consistency with the limits on baryonic isocurvature perturbations requires that the spectral index of adiabatic perturbations produced during inflation be very close to unity. In a variant of our scenario in which the scalar is a gauge singlet, the connection between the baryon to entropy ratio and the inflationary scale is lost, although the basic mechanism of baryogenesis remains applicable.

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I. INTRODUCTION

Two important numbers are measured in modern cosmology: the net baryon to entropy ratio $n_B/s \sim 10^{-10}$ (where n_B is the difference between the number density of baryons and antibaryons and s is the entropy density) and the amplitude of the angular power spectrum of the cosmic microwave background (CMB) temperature anisotropies, $\delta T/T \sim 10^{-5}$. These two observations *independently require* new physics input from both the particle physics sector and the pure cosmological sector. New particle physics input is required to explain the origin of the baryon asymmetry (this new particle physics must satisfy the three Sakharov conditions [1]). The CMB anisotropies could be a natural consequence of inflation [2]. Again, new particle physics appears to be required in order to generate inflation.

In the initial models of baryogenesis (see e.g. [3, 4] for reviews of baryogenesis), no connection was made between baryogenesis and inflation. A newer scenario for baryogenesis that establishes a connection with inflationary cosmology is the Affleck-Dine (AD) mechanism [5] or variants thereof. In this scenario, the required baryon asymmetry is generated via explicitly CP-violating and baryon number-violating decays of a new scalar field ϕ , a field which may be related to the field driving cosmological inflation. However, no link between the value of n_B/s and the amplitude of $\delta T/T$ was made.

Here, we study a simple variant of AD baryogenesis (proposed in [6]) which has the special features that no new CP-violating phases and baryon number violating interactions need to be introduced in the Lagrangian. Assuming that the observed cosmological fluctuations were generated during a period of inflation, we find that in our baryogenesis scenario the value of n_B/s is determined by the amplitude of $\delta T/T$ (modulo a Yukawa coupling constant). Given a reasonable lower bound on the initial value of the rolling scalar field ϕ yielding baryogenesis,

we deduce a *lower bound* for the baryon asymmetry given the observed amplitude of the CMB anisotropies. For reasonable values of the Yukawa coupling constant, the lower bound yields the correct order of magnitude for the baryon to entropy ratio. The connection between the two observational values holds if ϕ is a $SU(2)_L$ doublet. We also examine the case where ϕ is a gauge singlet, and in this case the relation between the two observational quantities is lost [18].

II. GENERAL CONSIDERATIONS

Our approach to baryogenesis relies on a simple variant of the AD mechanism that was recently introduced in [6]. Central to this scenario is a new complex scalar field ϕ which obtains a non-vanishing expectation value as a consequence of quantum fluctuations during inflation. If this field has motion in the phase direction, it acquires a charge which leads to CP-violation and resulting leptogenesis. Note that ϕ neither carries non-vanishing baryon or lepton charges, nor does it involve *CP*-violating couplings in the Lagrangian [19]. We assume that the mass m_ϕ of ϕ is smaller than the Hubble expansion rate H_I during inflation. After inflation ends and the Hubble rate $H(t)$ has fallen to m_ϕ the field begins to roll towards the minimum of its potential $V(\phi)$.

The charge asymmetry of ϕ which leads to leptogenesis is (see e.g. [4]) [20]

$$Q_\phi = Im(\phi^* \dot{\phi}). \quad (2.1)$$

This charge vanishes if the potential is a function of $\phi^* \phi$. Thus, it is essential for our mechanism that it has a different form, for example if it is a function of ϕ^2 (plus complex conjugates). We consider two possibilities, first the case when ϕ is a $SU(2)_L$ doublet, second when it is a singlet under this gauge group. In the first case (the case

studied in [6] and which will be the main focus here), the potential leads to a violation of gauge invariance.

Let us first consider the case when ϕ is a $SU(2)_L$ doublet. The key idea is to view the gauge symmetry-violating potential as the result of a symmetry breaking phase transition at a higher scale. Specifically, let us assume that at some large scale (say Λ) there is an extra complex scalar field χ which is charged under $SU(2)_L$. The interaction Lagrangian is taken to be

$$\mathcal{L} = y_\nu \bar{\nu}_L^i \nu_R^i \phi + V(|\phi|^2, |\sigma|^2, \phi^\dagger \sigma) + h.c. . \quad (2.2)$$

which is gauge-invariant. We assume that at some scale $\tilde{\Lambda} < \Lambda$ the field σ takes on a symmetry-breaking expectation value $\langle \sigma \rangle$, and that it is massive and hence decouples on scales $\ll \tilde{\Lambda}$. The resulting effective Lagrangian automatically breaks the original gauge symmetry spontaneously and contains terms like ϕ^2 [21]. The original potential $V(|\phi|^2, |\sigma|^2, \phi^\dagger \sigma)$ reduces to a low energy effective potential $V(|\phi|^2, |<\sigma>|^2, \langle \sigma \rangle \phi^\dagger)$. In this limit, it follows immediately that the rolling of ϕ leads to $Q_\phi \neq 0$. The case when ϕ is a singlet under $SU(2)_L$ will be discussed in a separate section since the Yukawa interaction term in (2.2) no longer can be used.

The Yukawa coupling in (2.2) generates a Dirac mass for the neutrinos. In a process analogous to what happens in models of leptogenesis, the Yukawa coupling leads to the decay of ϕ , in our case into Dirac neutrinos, producing an asymmetry in the left-handed leptons. This asymmetry is then converted to the required baryon asymmetry via electroweak sphalerons transitions [9]. Note that in this mechanism, the total lepton number is not violated and the asymmetry in left-handed neutrinos is compensated by a corresponding asymmetry in the right-handed neutrino sector.

In the following we consider the usual particle physics standard model (SM) with the addition of right handed neutrinos, and assume that the neutrinos, like any other fermion of the SM, obtain Dirac masses. The neutrino Yukawa coupling is small enough to prevent an equilibration between the left-handed and right-handed neutrinos (see [6] for a discussion). Note that one could also add a Majorana mass term for the right-handed neutrino ν_R without spoiling the mechanism.

In the first class of models, when ϕ is a $SU(2)_L$ doublet, we find a direct correlation between the produced baryon to entropy ratio and the amplitude of density perturbations generated during inflation. Indeed, we are able to provide a lower bound on n_B/s which is in quantitative agreement with observations.

On the other hand, in the second class of models, where ϕ is a singlet, the issue of gauge invariance is not a problem. The leading order interaction terms are dimension five non-renormalizable operators which describe the decay of ϕ . If we allow for all possible interaction terms, it turns out that a sufficient baryon asymmetry can still be produced, although no longer with a direct connection with the inflationary scale.

III. CONNECTION BETWEEN n_B AND H_I

To establish the connection between the generated baryon asymmetry and the scale of inflation, we first estimate the baryon to entropy ratio produced from the initial charge asymmetry Q_ϕ (given in (2.1)). The field ϕ decays when $\Gamma = H$ (where Γ is the total decay width), producing an asymmetry in the left-handed neutrinos (and similar asymmetry with opposite sign in the right-handed sector). The amount of asymmetry transferred to neutrinos is given by the charge evaluated at the decay time folded with the branching ratio, i.e.

$$\frac{(n_\nu - n_{\bar{\nu}})}{s} = \frac{Q_\phi}{s} \frac{\Gamma_\nu}{\Gamma}, \quad (3.3)$$

where s is the entropy density at the time $\Gamma \simeq H$, and Γ_ν is the decay rate into neutrinos. Generically, $m_\phi \gtrsim \Gamma$. Thus, at the time $H \simeq m_\phi$ when ϕ can begin to roll, it will immediately start oscillating as matter around its minimum, and its energy density gets diluted as a^{-3} (where $a(t)$ is the cosmological scale factor) until $H \simeq \Gamma$. Now, evaluating (3.3) at the decay temperature $T \simeq \sqrt{M_{pl}\Gamma}$ we obtain

$$\frac{n_B}{s} \approx Im[\phi^* \dot{\phi}] \frac{\Gamma_\nu}{\Gamma^{5/2} M_{pl}^{3/2}}. \quad (3.4)$$

For the decay rate of ϕ into neutrinos, we take the usual perturbative result

$$\Gamma_\nu \approx y_\nu^2 m_\phi / (8\pi), \quad (3.5)$$

and for the total decay rate we assume

$$\Gamma \approx m_\phi^3 / \phi^2. \quad (3.6)$$

The latter result can be obtained by taking the usual perturbative decay rate as in (3.5) with an initially arbitrary coupling constant g , and taking for g the maximal value for which the decay is kinematically allowed, assuming that the mass of the decay product obtains a contribution $g < \phi >$ from the ϕ field. Note that this upper limit coincides with the decay width used in the original AD model [5]. In that case the only possible decay was to a light particle which has no direct coupling to ϕ of the lighter final state, mediated via an intermediate heavy virtual state (to which a direct decay is forbidden) thereby, leading to a decay rate of the order $(\mathcal{O}(m_\phi^3/\phi^2))$. However, a direct decay is also still possible if the coupling constants for the relevant interactions are sufficiently small.

With the results of the previous paragraph to obtain the branching fraction, we can evaluate the baryon to entropy ratio given by (3.4). Since the field rolling takes place at small field values, the potential can be approximated by $V(\phi) \approx m_\phi^2 \phi^2 + h.c.$. Note that Γ depends on time since the amplitude of ϕ depends on time due to the red-shifting $-\phi \propto a^{-3/2} \propto H^{3/4}$ which means that at the decay time $\phi = \phi_0 (\Gamma/m_\phi)^{3/4}$ where, ϕ_0 is the initial field value when $H \gtrsim m_\phi$. The condition $\Gamma = H$ is

obtained for $H = \Gamma = m_\phi(m_\phi/\phi_0)^{4/5}$. Note that the energy density of the field ϕ is always sub-dominant as long as $m_\phi^2\phi^2 \gtrsim T^4$ or equivalently $H \gtrsim m_\phi(\phi_0/M_{pl})^4$. This situation is allowed as long as the field values are somewhat smaller than M_{pl} which is the limit of interest to us. If the field is light ($m_\phi < H_I$), its quantum fluctuations are determined by H_I . To a first approximation, in a particular Hubble volume, ϕ undergoes a random walk with time step H^{-1} and amplitude H (see e.g. [10] for a recent discussion). The inflationary expansion effectively homogenizes the value of ϕ in our patch of the universe. Note that in our case (for the quadratic potential), the real and imaginary components of the field are uncorrelated and hence, it is likely that both these components will independently take a value of order H_I . This leads to the relative phase to be of order unity. Therefore, in the oscillatory phase, the charge asymmetry is $\sim m|\phi_0|^2$ multiplied by the dilution factor.

As a lower bound on the value of ϕ in our Hubble patch we take $\phi_0 \gtrsim H_I$. This leads to a lower bound on the value of n_B/s :

$$\frac{n_B}{s} = y_\nu^2 \left(\frac{\phi_0}{M_{pl}} \right)^{3/2} \left(\frac{\phi_0}{m_\phi} \right)^{13/10} \gtrsim 10^{-9} y_\nu^2, \quad (3.7)$$

where we have made use of the observed amplitude of the CMB fluctuations to fix H_I . We conclude that in the case when ϕ is a doublet case, a realistic value for n_B/s comes directly from the amplitude of quantum fluctuations of ϕ during inflation. This is the main result of our analysis.

IV. CONSTRAINTS FROM ISOCURVATURE PERTURBATIONS

We examine here possible constraints on our baryogenesis scenario from the current limits on the amplitude of the isocurvature perturbations which the above scenario would produce.

It is well known that perturbations in two different fields (namely an inflaton χ field and our charged field ϕ) generically lead to isocurvature perturbations if the two fields decay into different types of particles. This may lead to observable consequences in the measurements of CMB anisotropies and Large Scale Structure. Present experiments are consistent with a purely adiabatic spectrum, and already put constraints on the amount of isocurvature perturbations which may be present. Moreover, it is expected that future experiments may be able to detect even small contributions of isocurvature perturbations, providing a valuable tool for the investigation of the early Universe.

To study the fluctuations, we expand the metric to linear order about a FRW background. In longitudinal gauge, the line element is (see [11] for a review)

$$ds^2 = (1 + 2\psi)dt^2 - a^2(t)(1 - 2\psi)\delta_{ij}dx^i dx^j, \quad (4.8)$$

where ψ is a function of space and time and contains the information about scalar metric fluctuations (we are

assuming the absence of anisotropic stress). The curvature perturbation ζ on a comoving hyper-surface (which measures the adiabatic fluctuation) is given by [11]

$$\zeta \equiv \psi + H \frac{\delta\rho}{\dot{\rho}} \quad (4.9)$$

(Note that this quantity is independent of gauge). Here, $\delta\rho$ is the perturbation of the total energy density. On super-Hubble scales and in the absence of isocurvature perturbations, ζ is a conserved quantity. Inflationary cosmology predicts an approximately scale-invariant spectrum of fluctuations. Thus, neglecting the small deviations from scale-invariance, the Fourier modes of ζ are given by

$$\zeta(k) = \frac{\sqrt{4\pi}}{\sqrt{\epsilon_{tot} M_{pl}}} A_k e_A(k), \quad (4.10)$$

where $A_k \equiv H/\sqrt{2k^3}$ (using the convention that co-moving wavenumbers are dimensionless), and $e_A(k)$ is a Gaussian random variable normalized to one [12]. Here,

$$\epsilon_{tot} \equiv \epsilon_\chi + \epsilon_\phi \text{ and } \epsilon_I \equiv \frac{M_{pl}^2}{16\pi} \left(\frac{\partial V/\partial\phi_I}{V} \right)^2 \quad (4.11)$$

is the slow roll parameter for the field ϕ_I (note that because of the small value of the mass m_ϕ , ϕ will be slowly rolling during inflation). In the above, $V(\phi, \chi)$ is the potential for the fields during inflation. When the presence of the field ϕ during inflation does not change this prediction (this is true for example if both fields roll with constant velocity during inflation [12]). This means that the inflationary adiabatic perturbation is not seeded by isocurvature perturbations and thus they are statistically uncorrelated.

Moreover, in our scenario, the energy density of the field ϕ is always sub-dominant with respect to the radiation which is produced by inflaton decay. As a consequence, after inflation, the perturbations in the field ϕ , and thus also the fluctuations in the energy density of its decay products, do not contribute appreciably to ζ [13]. This means that we use as value for ζ at late times, the usual expression as in (4.10).

Following the above arguments, the interesting quantity for us is the baryon to photon isocurvature perturbation

$$S_{B\gamma} \equiv \frac{\delta(n_B/s)}{n_B/s} = \frac{\delta\rho_B}{\rho_B} - \frac{3}{4} \frac{\delta\rho_\gamma}{\rho_\gamma}, \quad (4.12)$$

which is by construction a gauge invariant quantity. Alternatively, we can express (4.12) as [13]

$$S_{B\gamma} = 3(\zeta_B - \zeta_\gamma), \quad (4.13)$$

where $\zeta_i \equiv \psi - H \frac{\delta\rho_i}{\dot{\rho}_i}$. Now, since ζ_γ is a conserved quantity, it is equal to its initial value ζ_χ which is given at the end of inflation. For baryons, following [13], we introduce

the quantity $\tilde{\zeta}_B \equiv \psi - H \frac{\delta n_B}{n_B}$. For non-relativistic particles we have $\zeta_B = \tilde{\zeta}_B$. This holds for baryons after the QCD phase transition. Once the field ϕ decays, a non vanishing $\tilde{\zeta}_B$ is created leading to a relation $\zeta_B \simeq \zeta_\phi$. Moreover, at all times before the decay ζ_ϕ is a conserved quantity, as long as there is no energy transfer between ϕ and other fluids. This means that $S_{B\gamma}$ is directly related to ζ_ϕ and ζ_χ whose values are fixed by the inflationary phase. In particular [12, 15]

$$|S_{B\gamma}| \simeq |S_{\phi\chi}| \simeq 3 \frac{\sqrt{4\pi}}{M_{pl}} \frac{\sqrt{\epsilon_{tot}}}{\sqrt{\epsilon_\phi}\sqrt{\epsilon_\chi}} A_k |e_S(k)| , \quad (4.14)$$

where e_S is a normalized Gaussian random variable. The quantity which is constrained by experiments is the ratio of the amplitudes of $S_{B\gamma}$ and ζ , which has to be less than about three if the isocurvature and adiabatic perturbations are uncorrelated [17]. This translates into the constraint

$$\left| \frac{S_{B\gamma}}{\zeta} \right| \simeq \frac{3\epsilon_{tot}}{\sqrt{\epsilon_\chi\epsilon_\phi}} \lesssim 3 . \quad (4.15)$$

In fact, evaluating the slow-roll parameter ϵ_ϕ for our quadratic potential and using the CMB constraint on the overall amplitude of scalar perturbations [16] we obtain

$$\left| \frac{S_{B\gamma}}{\zeta} \right| \simeq 6\sqrt{\pi} \frac{(7 \cdot 10^{16} GeV)^4}{m_\phi^2 \phi_0 M_{pl}} \frac{\epsilon_{tot}^2}{\sqrt{\epsilon_\chi}} \lesssim 3 . \quad (4.16)$$

This implies that our scenario is viable only if ϵ_{tot} during inflation is a very small number, which in turn implies that the spectral index has to be very close to one.

V. UNBROKEN GAUGE SYMMETRY

Finally, let us present a model where the complex scalar ϕ is an $SU(2)_L$ singlet. In order to couple ϕ with neutrinos, one has to include higher dimension terms, for instance,

$$\mathcal{L}_{\phi H\nu} = \frac{\lambda_{ij}}{\Lambda} H \bar{\nu}_L^i \nu_R^j \phi + h.c. . \quad (5.17)$$

Here, Λ is some cutoff scale, and λ_{ij} are couplings of order unity. In this scenario, the four-particle interaction generates the lepton asymmetry, through the decay of the field ϕ . Let us now analyze the number n_B/s in this case. The decay width is suppressed by the five dimensional interaction term. Hence, in order to obtain the observed baryon to entropy ratio, we require a value of ϕ_0 which is larger than H_I . Since ϕ undergoes a random walk with amplitude H_I during inflation, it is easy to obtain larger values of ϕ_0 provided m_ϕ is substantially smaller than H_I (see e.g. [10] for a recent discussion).

Given a value for ϕ_0 , and making use of an estimate for the asymmetry of the decay rate into neutrinos during the process $\phi \rightarrow H\nu_L\nu_R$ which we take to be $\Gamma_\nu \approx m_\phi^3/\Lambda^2$, we obtain (following the same logic as in the third section)

$$\frac{n_B}{s} \approx \frac{\phi_0^{14/5} m_\phi^{7/10}}{M_{pl}^{3/2} \Lambda^2} . \quad (5.18)$$

Assuming $\Lambda \approx M_{pl}$, and for sufficiently large field values and masses, (5.18) can yield the correct value for n_B/s . If m_ϕ is smaller than H_I , we expect isocurvature perturbations to be again given by eq. (4.15) (as before, a small slow roll parameter is required to be consistent with observations).

VI. DISCUSSION AND CONCLUSIONS

To summarize, in the minimal model of baryogenesis proposed in [6], in which a rolling scalar field obtains a CP-asymmetry due to its initial conditions, and in which the decay of this field generates an asymmetry in the left-handed neutrino number density, we have shown that the baryon to entropy ratio is determined by the amplitude of CMB temperature fluctuations, thus connecting two observational numbers from cosmology which usually are considered not to be related. This relation holds if the scalar field is charged under $SU(2)_L$. More specifically, making use of the observed amplitude of the CMB anisotropies we obtain a lower bound on the baryon to entropy ratio which agrees quite well with the observed value. The exact value also depends on the magnitude of the neutrino Yukawa coupling.

We have also analyzed the constraints on our model coming from the observational limits on isocurvature perturbations. We find consistency provided that the adiabatic perturbations have a spectral index very close to the scale invariant one. Moreover, evidence for baryon isocurvature perturbations may be a signal for this model, giving information on the predicted baryon asymmetry. In order to completely calculate the baryon asymmetry, the knowledge of the Yukawa coupling of the new $SU(2)_L$ scalar with neutrinos is required. The link between n_B/s and $\delta T/T$ is lost if we consider the scalar field driving baryogenesis to be a gauge singlet (which we illustrated using a dimension five interaction).

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- [19] For earlier discussions on how to obtain cosmological CP-violation without explicit CP-violating phases in the Lagrangian see [3, 8].
- [20] Note that in the phase when ϕ is oscillating homogeneously in space, the charge density $Im(\phi^*\phi)$ turns out to be of the same order of the number density of the $k = 0$ quanta of ϕ which the field configuration corresponds to. During a phase of slow rolling, there is an extra factor which relates the number density to $Im(\phi^*\phi)$.
- [21] We are assuming that the vacuum expectation value of σ does not break the SM symmetries in any other term.